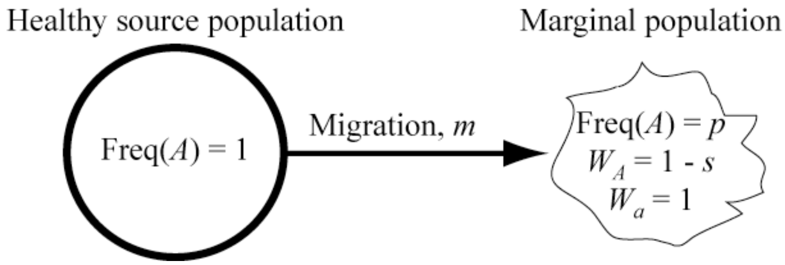
Problem 5.10: Species living at the edge of their natural range often fail to adapt to local conditions because of the constant inflow of migrants from the center of the range. Consider a haploid model of selection where selection in a marginal patch favors allele *a*: $W_A = 1 - s$ and $W_a = 1$. Adults migrate into the marginal patch from a more favorable area at rate *m*. We'll assume that these migrants all carry allele *A*, which is favored in the core habitat (Figure 5.7).

Evolution in a source and marginal habitat.



After migration, the frequency of the locally unfit allele, *A*, becomes $p(t+1) = (1-m)\,p' + m$, where *p*' is the frequency of allele *A* in the local population after selection but before migration:

$$p' = \frac{p(t)\,(1-s)}{p(t)\,(1-s) + (1-p(t))}$$

(assuming random mating). (a) Find the two equilibria of this model. (b) What conditions must hold for polymorphic equilibrium to be biologically valid? (c) Determine when allele *a* will disappear from the population when rare despite the fact that it is locally favored by examining the stability of the equilibrium at $\hat{p} = 1$.

Problem 5.13 (MODIFIED): Population size might be regulated by competition for suitable territories. Consider a large number of suitable territories or patches. At time *t*, a fraction, *p*, of these patches are occupied. Of the unoccupied sites, sites are recolonized at rate *m p*, which rises with the fraction of occupied patches. In addition, each occupied site suffers a risk of local extinction, at rate *e*, through catastrophic events such as fire or disease. These assumptions are consistent with the following differential equation in continuous time:

$$\frac{dp}{dt} = mp(1-p) - ep$$

(a) Find the two equilibria of this model. (b) Under what conditions is there a biologically valid equilibrium with the species present (i.e., when does *p* lie between 0 and 1)? (c) Given that the equilibrium in (b) is valid, when is it stable? (d) Is it possible for the fraction of occupied sites to overshoot the equilibrium?